P1 - Fund. de Cálculo (2014)

1)

a) $\lim_{x \to +\infty} \text{sen}\left(\frac{1}{x}\right) = \lim_{y \to 0+} \text{sen}(y) = 0$

b) $\lim_{x \to +\infty} x^2 \left(1 - \cos\left(\frac{1}{x}\right)\right) = \lim_{y \to 0+} \frac{1 - \cos y}{y^2}$

$= \lim_{y \to 0+} \frac{1 - \cos^2 y}{y^2(1 + \cos y)} = \lim_{y \to 0+} \frac{\text{sen}^2 y}{y^2} \cdot \frac{1}{(1 + \cos y)} = \frac{1}{2}$

2) Considere $h(x) = e^{-x} - \log_{10} x$.

É fácil ver que $h(x)$ é função contínua.

$h(1) = e^{-1} - \log_{10} 1 = e^{-1} > 0$

$h(10) = e^{-10} - \log_{10} 10 = e^{-10} - 1 < 0$

Pelo TVI, existe $c \in (1, 10)$ tq $h(c) = 0$, ou seja

$e^{-c} - \log_{10} c = 0$

Donde c é raiz de

$e^{-c} = \log_{10} c$

3) Se $x > 5$ temos $x \neq 0$ e $x \neq -2$.

Dividindo as desigualdades por $(x+2)$ temos:

$$\frac{4x - 1}{x} < f(x) < \frac{4x + 3}{x}$$

Como $\lim_{x \to +\infty} \frac{4x - 1}{x} = \lim_{x \to +\infty} \frac{4x + 3}{x} = 4$,

Pelo teorema do confronto temos:

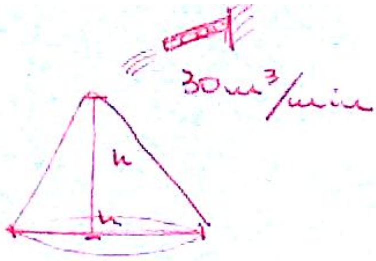
$$\lim_{x \to +\infty} f(x) = 4$$

4) $f'(x) = \dfrac{\frac{\cos(\sqrt{x})}{2\sqrt{x}} \cdot x - \text{sen}(\sqrt{x})}{x^2}$

$f'\left(\frac{\pi^2}{16}\right) = \dfrac{\cos\left(\frac{\pi}{4}\right) \cdot \frac{\pi}{8} - \text{sen}\left(\frac{\pi}{4}\right)}{\left(\frac{\pi^2}{16}\right)^2} =$

$= \frac{\sqrt{2}}{2}\left(\frac{\pi}{8} - 1\right) \frac{256}{\pi^4} = \boxed{\frac{128\sqrt{2}}{\pi^4}\left(\frac{\pi}{8} - 1\right)}$

5)



$$V = \pi r^2 \frac{h}{3} = \frac{\pi h^3}{12}$$

$$V(t) = \frac{\pi}{12} (h(t))^3$$

$$V' = \frac{\pi}{4} h^2 \cdot h'$$

$V' = 30$
$h = 10$ $\longrightarrow$ $3\cancel{0} = \frac{\pi}{4} 10\cancel{0} \cdot h'(t_0)$

$$\boxed{h'(t_0) = \frac{12}{10\pi} = \frac{6}{5} \cdot \frac{1}{\pi} \ m/min}$$